Sky radiance during a total solar eclipse: a theoretical model

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This paper describes a radiative transfer model for estimating the brightness or radiance of the sky during a total solar eclipse. The model is approximate; it only considers sunlight that diffuses into the umbra by first- and second-order scattering processes. It nevertheless correctly predicts the major observed features of the eclipsed sky including reddening of the horizon and lowering of zenith radiance over the normal day value by approximately 4 orders of magnitude. The calculated absolute zenith radiance at blue wavelength is about 20% lower than observed during the 1973 African solar eclipse. The model predicts, and observations confirm, that the zenith has the highest blue–red ratio (color temperature) and lowest brightness of any place in the sky during totality.

I. Introduction

Radiance of the sky during a total solar eclipse is mainly due to sunlight diffusing into the umbra by multiple-scattering processes. Airglow, light from the stellar background, light scattered by the interplanetary dust, and direct emission from the solar corona also contribute to sky radiance at totality, but except within a circular region of several solar radii from the sun, or at specific airglow emission wavelengths, these contributions are all small in comparison with multiple-scattered sunlight.

In this paper, I describe an approximate solution to the equations of transfer that considers the contribution to totality sky radiance caused from photons that have undergone at most two scattering processes in the earth's atmosphere. The neglect of light resulting from tertiary and higher order scattering is evidently a reasonable approximation since comparison of the theoretical sky radiance with the observed sky radiance during the 1973 African eclipse agrees with each other quite well.

II. Light-Scattering in an Eclipse Geometry

Calculations of the diffusion of photons from the dimly lit penumbra into the umbra could be carried through exactly by using Monte Carlo methods (Blattner *et al.*,[1] for instance). The model that was actually used to make the calculations reported in this paper is a greatly simplified model that only takes into account first- and second-order scattering processes in the atmosphere. The use of such a model, of course, depends on the validity of the assumption that third-order and higher-order scattering processes do not make significant contributions to total sky radiance. A certain justification for making the assumption is that modeled values of sky radiances agree to within about 15% to measured values of sky radiance during the 1973 solar eclipse in Africa. Nevertheless, there is a question as to how far the model can be extended and still give moderately accurate answers. A rough analysis indicates that errors probably would start to become intolerable if one attempted to use this simplified model for solar elevation angles less than about 30°.

Figure 1 illustrates the photon scattering processes that occur during totality. Note that primary-scattered sunlight ($n = 1$) illuminates the sky near the horizon (Ray 1), but only second-order (Ray 2) and higher order (Ray 3) scattering processes contribute to the sky illumination at higher elevation angles. To provide a scale for Fig. 1, the diameter of the umbra can vary from zero to a maximum possible value of about 110 km; it was 99-km diam for the 1973 African solar eclipse—the calculations have been made using the latter value.

As seen schematically in Fig. 1, photons entering the umbra pass almost horizontally through the atmosphere from the penumbra zone, which is itself only dimly illuminated by the partially blocked sun. The combination of low light levels in the near-penumbra and the heavy attenuation of the rays along their trajectory provides a rather small photon flux from all azimuths into the central umbra; these photons diffuse within the umbra by multiple scattering and cause the entire sky to be dimly lit.

The magnitude of the sky illumination, the sky color, and the distribution of radiance on the sky with angle would be expected to depend on such variables as the magnitude of the eclipse, the solar elevation angle, and on the amount and distribution of dust in the atmosphere since dust particles scatter light.

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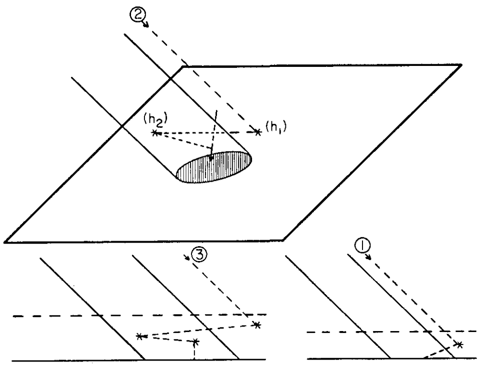

Fig. 1. The geometry of a total solar eclipse shown schematically in an isometric view in the top drawing and edge-on in the lower drawings. Light rays that undergo 1, 2, and 3 scattering processes are illustrated.

The radiance at the zenith caused by second-order scattering (see Ray 2 in Fig. 1) is calculated by integrating the appropriate contributions of light from all scattering points for second-order scattering in the eclipse geometry, i.e.,

$$I_{\text{zenith}}(\lambda) = F_0(\lambda) \int_{\phi=0}^{2\pi} \int_{h_2=0}^{\infty} \int_{h_1=0}^{\infty} \int_{r=r_0}^{\infty} T(h_1,h_2,r,\lambda) f_1(h_1,\lambda) \cdot f_2(h_2\lambda) \cdot \frac{L(r)}{r} drdh_1dh_2d\phi. \qquad (1)$$

The symbols have the following meaning:

$L(r)$ is the penumbra illumination function. It is zero at the edge of the umbra and 1.0 at the outer edge of the penumbra. It simply tells what fraction of the sun is covered by the moon at different distances from the center of the eclipse. For improved accuracy, solar limb darkening should be accounted for; I used values of this function listed by Outcalt and Meisel.[2]

$F_0(\lambda)$ is the extraterrestrial incoming solar flux at wavelength λ.

$T(h_1,h_2,r,\lambda)$ is the optical transmission of the light ray through the atmosphere along the indicated ray paths. h_2 is the height of the scattering point above the observer, h_1 is the height of the first scattering point in the penumbra, and r is the radius from the observed to the primary scattering point (see Fig. 1).

ϕ is an azimuth angle.

$f(h_i,\lambda)$ is the scattering function defined by the equation

$$f(h_i,\lambda) = \frac{\beta_M(\lambda,h_i)P_M(\theta) + \beta_D(\lambda,h_i)P_D(\theta)}{4\pi}, \qquad (2)$$

where β is a volume scattering coefficient, and P represents a scattering phase function. The subscripts M and D refer to scattering by molecules and dust. θ is the angle of scattering and is approximately equal to $\pi/2$ rad at both points 1 and 2.

In Eq. (1) the optical transmission function T is the product of the optical transmission along the three straight-line paths shown in Fig. 1, $T = T_{01} \cdot T_{12} \cdot T_{23}$, where

$$T_{ij} = \exp\left\{- \int_i^j \beta[h(l)]dl\right\}, \qquad (3)$$

and the integral is taken along the ray path l. It is absolutely necessary to include the curvature of the earth's atmosphere when calculating T_{12}.

In this model refraction of the ray was ignored, although it could have been included at the expense of making the calculations more difficult to perform. Its neglect has little effect on the results.

Equations (1)–(3) were solved numerically and applied to specific atmospheric models.

III. Discussion of the Radiative-Transfer Model

There is a danger in overmodeling. The objective in this work has been to understand the basic processes that contribute to the observed sky brightness and color during totality. If a simplified treatment can be shown to give results that agree reasonably well with observations (and it does), one can easily carry out tests to find the sensitivity of the sky radiance to changes in parameters.

There is no use introducing detailed complicated expressions for the atmospheric parameters into the model since this is an approximate model whose aim is to provide no more than about a factor of 2 accuracy. I therefore have used a simple exponential equation (with constant scale height) to describe the vertical distribution of molecular density with height. It also seems reasonable to suppose that dust is mixed into the atmosphere exponentially decreasing with altitude with about a 2-km scale height. The dust scattering phase function used in the calculations was taken at 0.1 and supposed not to vary over the scattering angles, which were usually close to about $\pi/2$ rad. The values of scattering phase function and aerosol optical depths were determined on the day of the eclipse at Loiyengalani, Kenya from spectral measurements of atmospheric transmission and measurements of sky brightness.

Surface reflection effects have been ignored in the model, although rough calculations, assuming that one-fourth of the downwelling flux is reflected up then scattered down, show that surface reflections may increase the sky brightness by roughly 10% or so in the blue.

This is a scalar model and does not include polarization. The presence of clouds in the atmosphere and the necessity of including high-order scattered light would confuse any attempt made at calculating sky polarization, although polarization could in principle be accounted for in a Monte Carlo solution to the transfer equations.

IV. Results and Discussion

Figure 2 shows how the radially inward sky intensity (in arbitrary units) from a ring-shaped scattering volume in the penumbra (h_1 to $h_1 + dh_1$, r to $r + dr$) would look to an observer (located in the center of the umbra) at the surface [$h_2 = 0$, Fig. 2(a)] and at $h_2 = 16$-km [Fig. 2(b)] altitude. The ordinate is the altitude of the scattering point in the penumbra (h_1) (km); the parameter in the curves is r (km). The main contribution of light

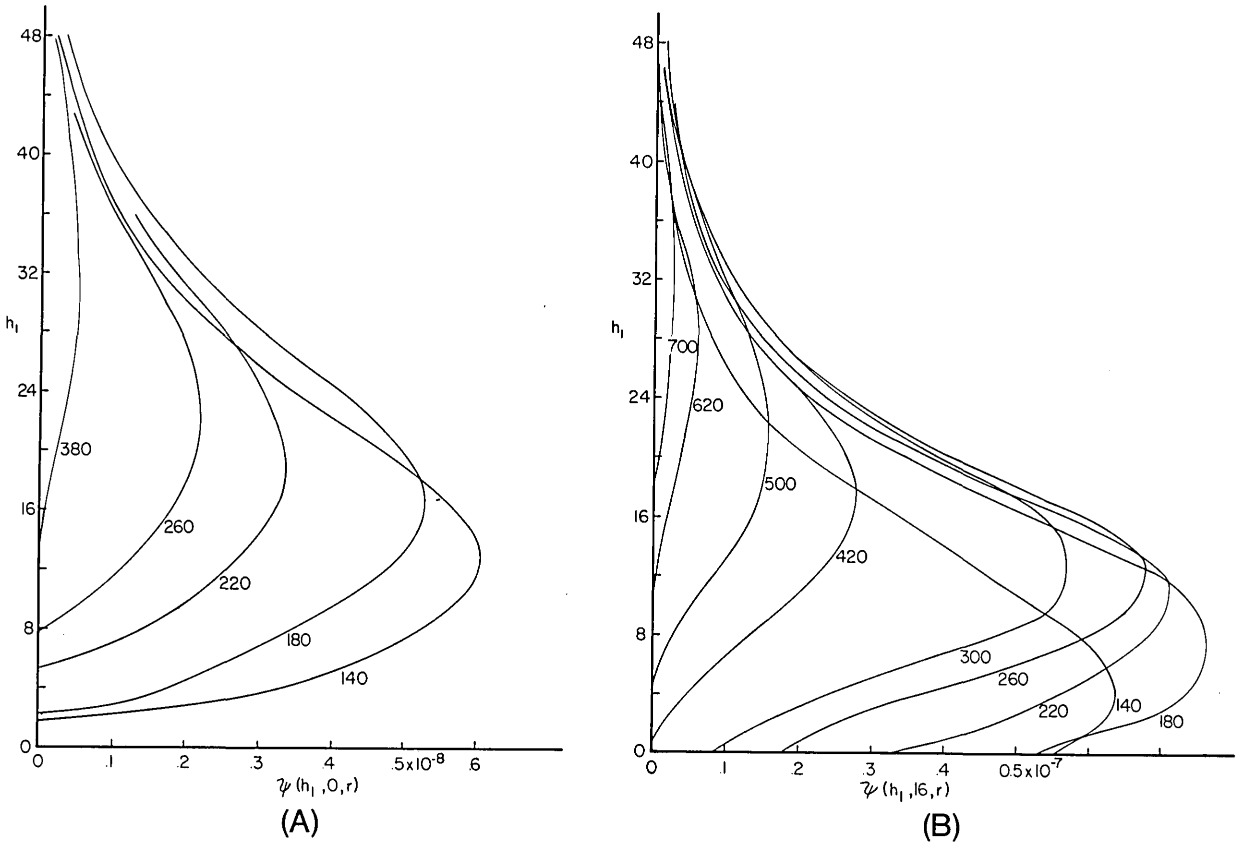

Fig. 2. The distribution of illumination (in arbitrary units) scattered from ring-shaped volume elements in the penumbra, at altitude h_1, and at radial distances r (shown as the parameter in kilometers) as viewed by an observer at the center of the umbra from the surface (A) and from an altitude of 16 km (B).

is due to scattering in the upper troposphere and lower stratosphere at altitudes from 8 km to 10 km. Notice that the brightness of the ring of light around the observer decreases as the emitting radius r increases to several hundred kilometers, even though the direct illumination by the sun is increasing with distance r (in the penumbra). The reason for the decrease is that the incoming light from a distant ring undergoes attenuation along its path which, at large distances, more than compensates for the increased direct illumination of the outer rings. This effect is more clearly shown in Fig. 3(B) where the vertically integrated radiance emanating from cylindrical rings of thickness dr and radius r is plotted as a function of r on a normalized scale. A ring with radius smaller than the umbra radius is not directly illuminated and therefore does not contribute anything.

As seen in Fig. 3(B), incoming light from rings near 175-km radius are the major contributors to the sky radiance. Clouds, and especially high clouds, anywhere within a radius of about 200 km or 300 km will block the almost-horizontal rays from the scattering ring and probably cause a darker eclipse.

In Fig. 3(A), the vertical distribution of incoming illuminating flux over the observer is shown for different scattering rings. Notice that the light of the sky during eclipses originates mainly in the upper troposphere. Calculations made to determine sky darkening with altitude show that a 40% reduction in brightness occurs at 11.2 km (37 KFT), the cruising altitude of modern jets.

Figure 4 shows the wavelength dependence of the zenith sky radiance predicted by the model and several measured values of sky radiance during the 1973 African eclipse.[3] The measured radiance agrees rather well with the theory, especially at the blue wavelengths. Presumably, the difference between the observed and calculated curves is caused from neglecting high-order scattering, which apparently contributes about 15% of the total sky radiance in the midvisible.

Sensitivity of sky radiance on atmospheric and geometric parameters: Evaluation of the model for different combinations of variables indicates the following:

(a) The effect of atmospheric dust: adding dust to the atmosphere darkens an eclipse. However, dust in the lower troposphere has a smaller effect on sky radiance than might at first be imagined. This is because a high proportion of the incoming photons have scattered at altitudes about 10 km. Removing the dust from the model, ($\tau_D = 0.2, H = 2$ km) increases the zenith sky radiance by 10% at 400 mm, 25% at 500 mm, and 35% at 600 mm. Dust in the stratosphere, however, would darken the eclipsed sky.

(b) The effect of varying solar elevation angle:

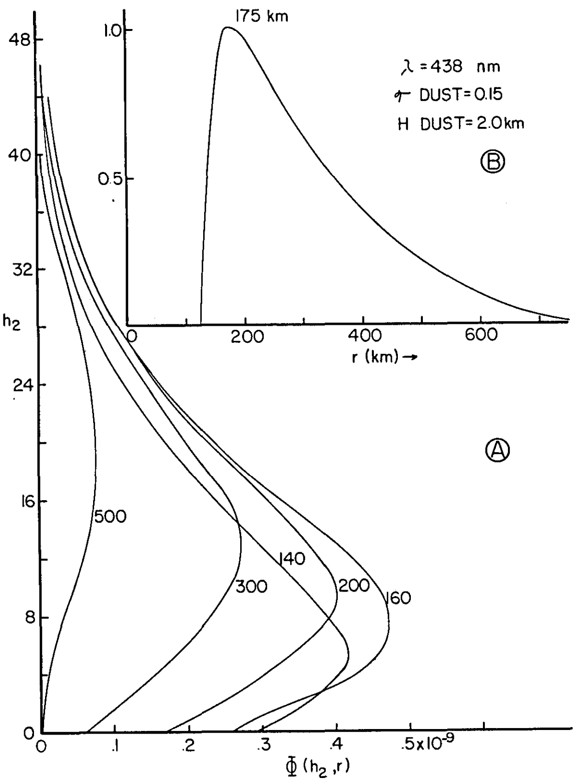

Fig. 3. (A) Vertical distribution of downwelling radiance over the center of the eclipse from different cylindrical-shaped volume elements at radial distance r (km) and radial thicknesses equal to 40 km. (B) The vertically integrated and azimuthally integrated contribution to zenith sky radiance as a function of radial distance from the center of eclipse.

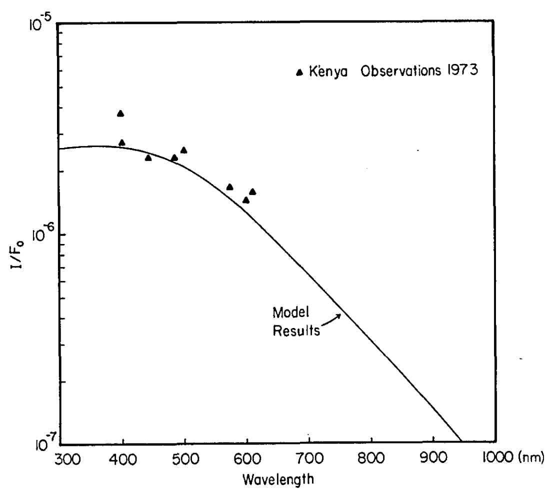

Fig. 4. Wavelength-dependence of zenith sky radiance computed by the scattering model for conditions during the 1973 African solar eclipse in Northern Kenya (solar elevation angle was 37°). Observations are indicated by triangles—they are accurate to about 10%.

Evaluation of the model indicates that the zenith darkens as the solar zenith angle increases. Typical values of the ratio $R = I(z_0)/I(0)$, where z_0 is the sun's zenith angle, are $z_0 = 20°, R = 0.93$; $z_0 = 30°, R = 0.83$; $z_0 = 40°, R = 0.66$. Assumptions used in this model begin to break down at larger solar zenith angles.

(c) The distribution of radiance on the sky: The model indicates, and observations confirm, that the zenith is the darkest place in the sky during totality. Sky radiance, as predicted from the model, can be approximated quite well by $I_{sky}(z) = I_{sky}(0)\cdot\sec(z)$ for $0 < z \gtrsim 50°$ (where z is the viewing zenith angle). There is practically no dependence of sky radiance on azimuth (provided the solar elevation angle is larger than 20°), except near the horizons which are slightly brighter at ±90° from the sun than at 0° and 180°. A discussion of eclipsed sky brightness near the horizon is given in a paper by Gedzelman.[4]

(d) Color: For a given eclipse the red/blue ratio of the skylight is smallest at the zenith, but remains almost constant down to an elevation angle of about 30°. At lower viewing angles the sky reddens, in accord with the African eclipse observations. Although the near-zenith sky temperature increased (became bluer) in going from noneclipsed to eclipsed sun conditions during the 1973 African eclipse, this will not always necessarily occur. The rather large values of blue-to-red reported during eclipses[5] reflects the fact that the scattered light eminates mainly from high altitudes where dust loading is small and Rayleigh-scattering dominates. The normal noneclipsed day sky intensity, on the other hand, is often reddened by dust scattering in the low atmosphere. Thus the sky color can, and does, shift to the blue during totality. The shift of sky color toward the blue would be expected to be larger and more dramatic when the lower atmosphere is charged with dust.

(e) Eclipse magnitude: Figure 5 shows the sensitivity of zenith sky brightness at three wavelengths to changes in the umbra radius. The curves in Fig. 5 indicate that short-duration eclipses (small radius) have brighter skies than long-duration eclipses. The blue/red color index also increases slightly for eclipses with small-diameter shadows.

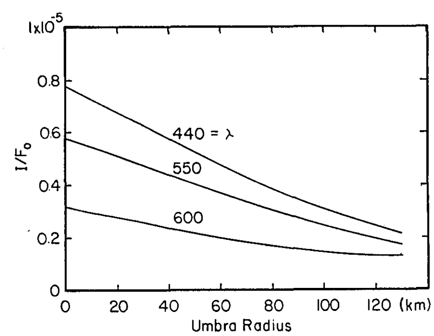

Fig. 5. Zenith sky radiance I/F_0 (per sr) at wavelengths 440-nm, 550-nm, and 600-nm wavelengths, as a function of the umbra radius.

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Glenn E. Shaw on the roof of the Geophysical Institute in Alaska