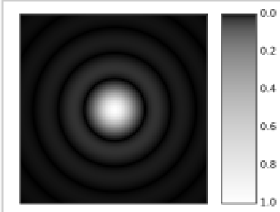
Airy disk

From Wikipedia, the free encyclopedia
(Redirected from Airy disc)



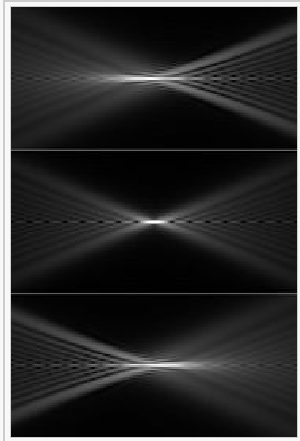
Computer-generated image of an Airy disk. The gray scale intensities have been adjusted to enhance the brightness of the outer rings of the Airy pattern.

In optics, the Airy disk (or Airy disc) and Airy pattern are descriptions of the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light.

The diffraction pattern resulting from a uniformly-illuminated circular aperture has a bright region in the center, known as the Airy disk which together with the series of concentric bright rings around is called the Airy pattern. Both are named after George Biddell Airy, who first described the phenomenon. The diameter of this pattern is related to the wavelength of the illuminating light and the size of the circular aperture.

The most important application of this concept is in cameras and telescopes. Due to diffraction, the smallest point to which one can focus a beam of light using a lens is the size of the Airy disk. Even if one were able to make a perfect lens, there is still a limit to the resolution of an image created by this lens. An optical system in which the resolution is no longer limited by imperfections in the lenses but only by diffraction is said to be diffraction limited.

The Airy disk is of importance in physics, optics, and astronomy.



Longitudinal sections through a focused beam with (top) negative, (center) zero, and (bottom) positive spherical aberration. The lens is to the left.

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Size [edit]

Far away from the aperture, the angle at which the first minimum occurs, measured from the direction of incoming light, is given the empirical formula:

$$\sin\theta = 1.22\frac{\lambda}{d}$$

where λ is the wavelength of the light and *d* is the diameter of the aperture. The Rayleigh criterion for barely resolving two objects is that the center of the Airy disk for the first object occurs at the first minimum of the Airy disk of the second. This means that the angular resolution of a diffraction limited system is given by the same formula.

Examples [edit]

Cameras [edit]

If two objects imaged by a camera are separated by an angle small enough that their Airy disks on the camera detector start overlapping, the objects can not be clearly separated any more in the image, and they start blurring together. Two objects are said to be *just resolved* when the maximum of the first Airy pattern falls on top of the first minimum of the second Airy pattern (the Rayleigh criterion).

Therefore the smallest angular separation two objects can have before they significantly blur together is given as stated above by

$$\sin\theta = 1.22\ \frac{\lambda}{d}$$

Thus, the ability of the system to resolve detail is limited by the ratio of λ/d. The larger the aperture for a given wavelength, the finer the detail which can be distinguished in the image.

Since θ is small we can approximate this by

$$\frac{x}{f} = 1.22\,\frac{\lambda}{d}$$

where x is the separation of the images of the two objects on the film and f is the distance from the lens to the film. If we take the distance from the lens to the film to be approximately equal to the focal length of the lens, we find

$$x = 1.22\,\frac{\lambda f}{d}$$

but $\frac{f}{d}$ is the f-number of a lens. A typical setting for use on an overcast day would be $f/8$.[1] For blue visible light, the wavelength λ is about 420 nanometers.[2] This gives a value for x of about 0.004 mm. In a digital camera, making the pixels of the image sensor smaller than this would not actually increase image resolution.

The human eye

[edit]

The fastest f-number for the human eye is about 2.1,[3] corresponding to a diffraction-limited point spread function with approximately 1 μm diameter. However, at this f-number, spherical aberration limits visual acuity, while a 3 mm pupil diameter (f/5.7) approximates the resolution achieved by a the human eye.[4] The maximum density of cones in the human fovea is approximately 170,000 per square millimeter,[5] which implies that the cone spacing in the human eye is about 2.5 μm, approximately the diameter of the point spread function at f/5.

Focused laser beam

[edit]

A circular laser beam with uniform intensity across the circle (a flat-top beam) focused by a lens will form an Airy disk pattern at the focus. The size of the Airy disk determines the laser intensity at the focus.

Conditions for observation

[edit]

Light from a uniformly illuminated circular aperture (or from a uniform, flattop beam) will exhibit an Airy diffraction pattern far away from the aperture due to Fraunhofer diffraction (far-field diffraction).

The conditions for being in the far field and exhibiting an Airy pattern are: the incoming light illuminating the aperture is a plane wave (no phase variation across the aperture), the intensity is constant over the area of the aperture, and the distance R from the aperture where the diffracted light is observed (the screen distance) is large compared the aperture size, and the radius a of the aperture is not too much larger than the wavelength λ of the light. The last two conditions can be formally written as $R > a^2 / \lambda$.

In practice, the conditions for uniform illumination can be met by placing the source of the illumination far from the aperture. If the conditions for far field are not met (for example if the aperture is large), the far-field Airy diffraction pattern can also be obtained on a screen much closer to the aperture by using a lens right after the aperture (or the lens itself can form the aperture). The Airy pattern will then be formed at the focus of the lens rather than at infinity.

Hence, the focal spot of a uniform circular laser beam (a flattop beam) focused by a lens will also be an Airy pattern.

In a camera or imaging system an object far away gets imaged onto the film or detector plane by the objective lens, and the far field diffraction pattern is observed at the detector. The resulting image is a convolution of the ideal image with the Airy diffraction pattern due to diffraction from the iris aperture or due to the finite size of the lens. This leads to the finite resolution of a lens system described above.

Mathematical details

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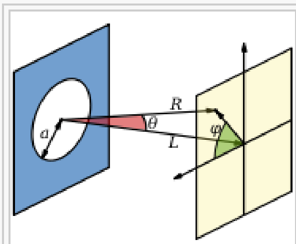
The intensity of the Fraunhofer diffraction pattern of a circular aperture (the Airy pattern) is given by:

$$I(\theta) = I_0 \left(\frac{2 J_1(ka \sin \theta)}{ka \sin \theta} \right)^2 = I_0 \left(\frac{2 J_1(x)}{x} \right)^2$$

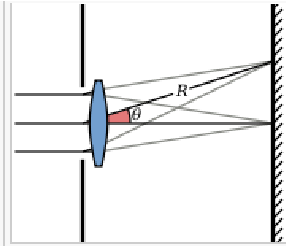
where I_0 is the maximum intensity of the pattern at the airy disc center, J_1 is the Bessel function of the first kind of order one, $k = 2\pi / \lambda$ is the wavenumber, a is the radius of the aperture, and θ is the angle of observation, i.e. the angle between the axis of the circular aperture and the line between aperture center and observation point. $x = ka \sin\theta = \frac{2\pi a}{\lambda}\frac{q}{R} = \frac{\pi q}{\lambda N}$, where q is the radial distance from the optics axis in the observation (or focal) plane and $N = R / d$ (d=2a is the aperture diameter, R is the observation distance) is the f-number of the system.



Diffraction from a circular aperture. The Airy pattern is observable when $R > a^2 / \lambda$ (i.e. in the far field)

If a lens after the aperture is used, the Airy pattern forms at the focal plane of the lens, where R = f (f is the focal length of the lens). Note that the limit for $\theta \longrightarrow 0$ (or for $x \longrightarrow 0$) is $I(0) = I_0$.

The zeros of $J_1(x)$ are at $x = ka \sin\theta \approx 0, 3.8317, 7.0156, 10.1735, 13.3268, 16.4706 \ldots$. From this follows that the first dark ring in the diffraction pattern occurs where



Diffraction from an aperture with a lens. The far field image will (only) be formed at the screen one focal length away, where R=f (f=focal length). The observation angle θ stays the same as in the lensless case.

$$\sin\theta = \frac{3.83}{ka} = \frac{3.83\lambda}{2\pi a} = 1.22\frac{\lambda}{2a} = 1.22\frac{\lambda}{d}.$$

The radius q_1 of the first dark ring on a screen is related to θ by

$q_1 = R\sin\theta$

where R is the distance from the aperture. The half maximum of the central Airy disk (where $J_1(x) = 1/2$) occurs at $x = 1.61633...$; the 1/e^2 point (where $J_1(x) = 1/e^2$) occurs at $x = 2.58383...$, and the maximum of the first ring occurs at $x = 5.13562...$.

The intensity I_0 at the center of the diffraction pattern is related to the total power P_0 incident on the aperture by[6]

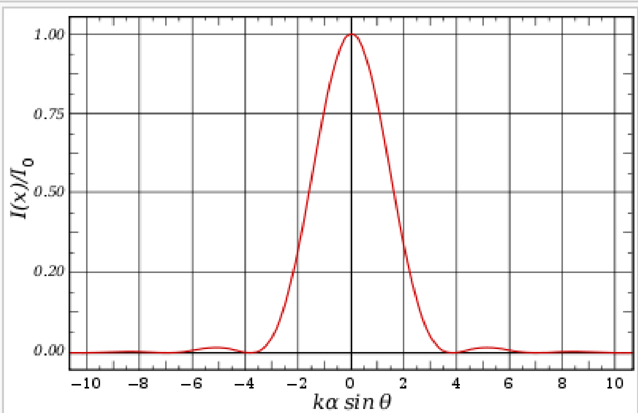
$$I_0 = \frac{\mathrm{E}_A^2 A^2}{2R^2} = \frac{P_0 A}{\lambda^2 R^2}$$

where E is the source strength per unit area at the aperture, A is the area of the aperture ($A = \pi a^2$) and R is the distance from the aperture. At the focal plane of a lens, $I_0 = (P_0 A) / (\lambda^2 f^2)$. The intensity at the maximum of the first ring is about 1.75% of the intensity at the center of the Airy disk.

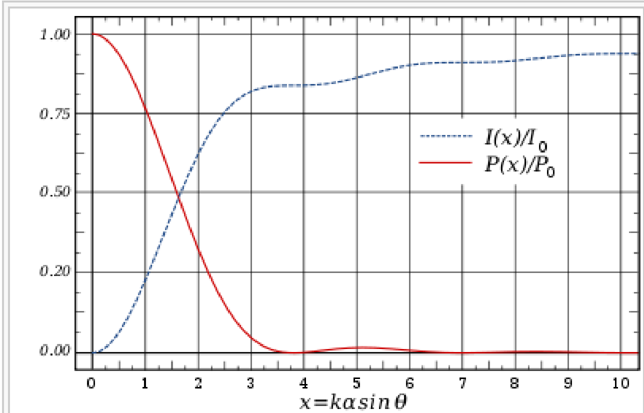
The expression for $I(\theta)$ above can be integrated to give the total power contained in the diffraction pattern within a circle of given size:

$$P(\theta) = P_0[1 - J_0^2(ka\sin\theta) - J_1^2(ka\sin\theta)]$$

where J_0 and J_1 are Bessel functions. Hence the fractions of the total power contained within the first, second, and third dark rings (where $J_1(ka\sin\theta) = 0$) are 83.8%, 91.0%, and 93.8% respectively.[7]



The Airy Pattern on the interval $ka\sin\theta$ = [−10, 10]



The encircled power graphed next to the intensity.

Approximations

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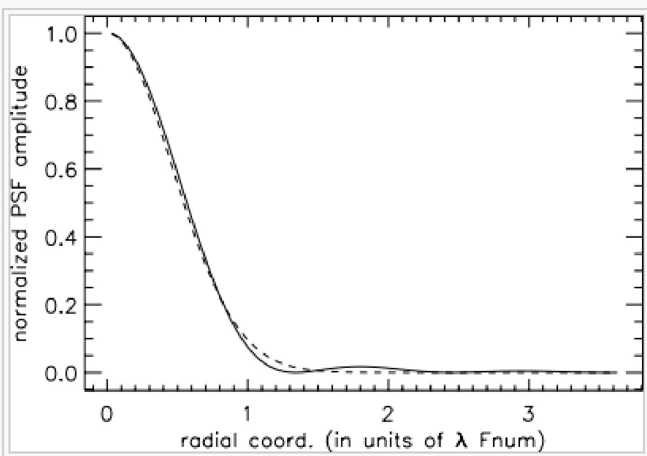
Because the Airy pattern is expressed in terms of Bessel functions, it can be difficult to find convenient analytical expressions for related quantities, such as the root mean square (RMS) spotsize. A common method is to ignore the relatively small outer rings of the Airy pattern and to approximate the central lobe with a Gaussian profile, such that

$$I(x) \approx I_0' \exp\left(\frac{-x^2}{2w^2}\right) \ ,$$

where I'_0 is the irradiance at the center of the pattern, and w is the Gaussian width. If we equate the peak amplitude of the Airy pattern and Gaussian profile to be equal, i.e. $I'_0 = I_0$, and find the value of w giving the optimal approximation[8] to the pattern, we obtain

$$w \approx 0.42\lambda f/D \ ,$$

where f is the focal length of the imaging system and D the diameter of the entrance pupil. If, on the other hand, we wish to



A radial cross-section through the Airy pattern (solid curve) and its Gaussian profile approximation (dashed curve). The abscissa is given in units of the wavelength λ times the f-number of the optical system.

enforce that the Gaussian profile has the same volume as does the Airy pattern, then this becomes

$$w \approx 0.44 \lambda f/D\ .$$

In optical aberration theory, it is common to describe an imaging system as *diffraction-limited* if the Airy disk radius is larger than the RMS spotsize determined from geometric ray tracing (see Optical lens design). Since the RMS spotsize is equivalent to the standard deviation of a function, the Gaussian profile approximation provides a convenient means of comparison: here the RMS spotsize is just the Gaussian width parameter, w. And, using the approximation above shows that the RMS spotsize of the Airy disk itself is about one-third that of the Airy disk radius, i.e. $0.42\lambda f / D$ as opposed to $1.22\lambda f / D$. Without resorting to approximation, since the outer lobes contribute significantly to the variance integral, the Airy pattern's true RMS spotsize is undefined (i.e. infinite).

Obscured Airy pattern

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Similar equations can also be derived for the obscured Airy diffraction pattern[9][10], which is the diffraction pattern from an annular aperture or beam, i.e. a uniform circular aperture (beam) obscured by a circular block at the center.

$$I(\theta) = \frac{I_0}{(1-\epsilon^2)^2}\left(\frac{2J_1(x)}{x} - \frac{2\epsilon J_1(\epsilon x)}{x}\right)^2$$

where ε is the annular aperture obscuration ratio, or the ratio of the diameter of the obscuring disk and the diameter of the aperture (beam). $(0 \le \epsilon < 1)$, and x is defined as above: $x = ka\sin(\theta) \approx \frac{\pi R}{\lambda N}$ where R is the radial distance in the focal plane from the optical axis, λ is the wavelength and N is the f-number of the system. The encircled energy (the fraction of the total energy contained within a circle of radius R centered at the optical axis in the focal plane) is then given by:

$$E(R) = \frac{I_0}{(1-\epsilon^2)^2}\left(1 - J_0^2(x) - J_1^2(x) + \epsilon^2\left[1 - J_0^2(\epsilon x) - J_1^2(\epsilon x)\right] - 4\epsilon\int_0^x \frac{J_1(t)J_1(\epsilon t)}{t}\,dt\right)$$

For $\epsilon \longrightarrow 0$ the formulas reduce to the unobscured versions above.

Comparison to Gaussian beam focus

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A circular laser beam with uniform intensity profile, focused by a lens, will form an Airy pattern at the focal plane of the lens. The intensity at the center of the focus will be $I_0{,}Airy = (P_0 A) / (\lambda^2 f^2)$ where P_0 is the total power of the beam, $A = \pi D^2 / 4$ is the area of the beam (D is the beam diameter), λ is the wavelength, and f is the focal length of the lens.

A Gaussian beam with $1 / e^2$ diameter of D focused through an aperture of diameter D will have a focal profile that is nearly Gaussian, and the intensity at the center of the focus will be 0.924 times $I_0{,}Airy$.[10]

See also

[edit]

- Fraunhofer diffraction
- Amateur astronomy
- Point spread function

Notes and references

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1. ^ See Sunny 16 rule.
2. ^ See cone cells for sensitivy of S cone cells.
3. ^ Hecht, Eugene (1987). *Optics* (2nd ed. ed.). Addison Wesley. ISBN 0-201-11609-X. Sect. 5.7.1
4. ^ *Optical System Design*. McGraw-Hill Professional. 2000. ISBN 0071349162.
5. ^ "Eye Receptor Density". Retrieved on 2008-09-20.
6. ^ E. Hecht, *Optics*, Addison Wesley (2001)
7. ^ M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, New York, 1965)
8. ^ Zhang et al., *Optics Express*, 15, 1543-1552 (2007): http://www.opticsexpress.org/abstract.cfm?uri=oe-15-4-1543
9. ^ Rivolta, *Applied Optics*, 25, 2404 (1986).
10. ^ ***a*** ***b*** Mahajan, *J. Opt. Soc. Am. A*, 3, 470 (1986).

External links

[edit]

- "Concepts and Formulas in Microscopy: Resolution" by Michael W. Davidson, *Nikon MicroscopyU* (website).
  - "Image Formation: Numerical Aperture and Image Resolution" (Interactive Java Tutorial) by Kenneth R. Spring, Brian O. Flynn, and Michael W. Davidson, *Molecular Expressions* (website), June 15, 2006.
  - "Image Formation: Airy Pattern Formation" (Interactive Java Tutorial) by Kenneth R. Spring, Brian O. Flynn, and Michael W. Davidson, *Molecular Expressions* (website), June 15, 2006.
- "Diffraction from a Circular Aperture" by Paul Padley, *Connexions* (website), November 8, 2005. – Mathematical details to derive the

above formula.

- "The Airy Disk: An Explanation Of What It Is, And Why You Can't Avoid It", *Oldham Optical UK*.

Categories: Physical optics | Diffraction