博士学位论文摘要选登

自旋致密双星后牛顿哈密顿系统轨道动力学数值研究

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由中子星或黑洞构成的旋转致密双星后牛顿哈密顿系统属于相对论二体问题，该系统不但含有丰富的共振和混沌等动力学现象，而且成为探测引力波的理想天然波源．引力体的轨道动力学性质会在引力波中得到反映．因此，实际天体的混沌性既可能是对引力波探测的挑战，又可望是获得观测效应的机遇．本学位论文正是在这样的国际学术氛围下数值研究旋转致密双星后牛顿保守哈密顿动力学问题．

基于最小二乘法原理我们构造了单和双标度因子等几种流形改正方法，分别对旋转致密双星后牛顿保守哈密顿系统中的总能量、总角动量 3 个分量和 2 个自转矢量长度共 6 个运动积分实施校正．以 5 阶 Rung-Kutta 方法为基准，研究后牛顿项、旋转与轨道耦合项、旋转与旋转耦合项以及轨道类型对这些流形改正方法和 Nacozy 方法的数值性能影响．当仅考虑 Kepler 轨道部分时，所有的流形改正方法几乎都具有相同的校正效果．若考虑纯轨道部分至 3 阶后牛顿时，各种流形改正方法会有明显不同的校正效果．对于旋转情形若采用一些省时措施则不会增加很多额外的计算时间，从而保证了数值计算效率和 Lyapunov 意义下的稳定性．数值发现混沌轨道比有序轨道更有利于发挥流形改正方法的效果，还证实了同时稳定总能量和总角动量的双标度因子法具有最佳的校正效果．流形改正避免了数值误差引起的虚假混沌，所得动力系统定性分析的结果更可靠．详见 Phys. Rev. D 81, 104037 (2010).

Lubich 等在 Phys. Rev. D 81, 104025 (2010) 中用非正则的辛算法数值求解致密双星后牛顿哈密顿系统，然而，他们构造的半隐 Euler 嵌入的混合辛积分器是有问题的，因为半隐 Euler 嵌入法比隐中点嵌入法的稳定性要差很多．我们改进了他们的工作，考虑利用由伍歆和谢懿所设计的正则旋转变量 (Phys. Rev. D 81, 084045 (2010)) 和 2 阶隐中点法以对称组合方式嵌套构成 2 阶和 4 阶混合积分器．即先将哈密顿分解为 Kepler 主要部分和摄动部分；再给出 Kepler 部分的解析解，并利用隐中点法获得摄动部分的数值解；然后将解析解和数值解对称组合成辛积分器．数值实验表明哈密顿摄动分解的 2 阶混合积分器比哈密顿不分解的隐中点法的精度要好很多，计算效率也相当．发现最优化的 4 阶混合辛算法在精度上要明显优越于 2 阶混合辛积分器，但需要增加一点额外的计算时间；还发现轨道的混沌性能促进迭代的快速收敛，提高辛方法的计算效率．辛算法的能量积分误差无长期变化，可确保从引力波获得的动力学信息更准确可靠．详见 Phys. Rev. D 82, 124040 (2010).

借助流形改正的 Rung-Kutta 算法或辛积分器以及伍歆等两粒子法的快速 Lyapunov 指标 (Phys. Rev. D 74, 083001 (2006)) 对旋转致密双星后牛顿哈密顿系统的相空间全局结构扫描都一致表明轨道动力学的跃迁与动力学参量、初始条件和初始旋转变量的全面综合有关，无法获得单个参数或初始条件的变化而引起动力学跃迁的普适规律．这的确是对伍歆和谢懿工作 (Phys. Rev. D 77, 103012 (2008)) 的有力支持．还证实混沌旋转致密双星辐射的引力波具有混沌特征．

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Numerical Investigations of Post-Newtonian Hamiltonian Dynamics for Spinning Compact Binaries

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Spinning compact binaries, consisting of neutron stars or black holes, not only have rich dynamic phenomena of resonance and chaos, but also are the most promising source for detecting gravitational waves. There should be a certain relation between the dynamics of the gravitational bodies and the gravitational waveforms.

Based on the least-squares correction, several manifold correction schemes like the single-scaling method and the dual-scaling method are designed to suppress numerical errors from 6 integrals of motion in a conservative post-Newtonian (PN) Hamiltonian of spinning compact binaries. Taking a fifth order Runge-Kutta algorithm as a basic integrator, we wonder whether the PN contributions, the spin effects, and the classification of orbits exert some influences on these correction schemes and the Nacozy's approach. It is found that they are almost the same in correcting the integrals for the pure Kepler problem. Once the third-order PN contributions are added to the pure orbital part, there are explicit differences of correction effectiveness among these methods. As an interesting case, the efficiency of correction is better for chaotic eccentric orbits than for quasicircular regular ones. In all cases tested, the new momentum-position dual-scaling scheme does always have the optimal performance. It costs a little but not much expensive additional computational cost when the spin effects exist, and several time-saving techniques are used. The corrected numerical results are more accurate than the uncorrected ones, so that chaos from the numerical errors can be avoided. See Phys. Rev. D 81, 104037 (2010) for more details.

Lubich et al. (Phys. Rev. D 81, 104025 (2010)) presented a noncanonically symplectic integrator for the PN Hamiltonian of a spinning compact binary. However, the Euler mixed integrator is problematic because of its bad numerical stability. We improved the work by constructing the second-order and the fourth-order fixed symplectic integrators, where the second-order symplectic implicit midpoint rule and its symmetric compositions are together used to integrate a PN Hamiltonian with the canonical spin variables of Wu and Xie (Phys. Rev. D 81, 084045 (2010)). Many numerical tests show that the mixed leapfrog integrator is always superior to the midpoint rule in the accuracy, while both of them are almost equivalent in the computational efficiency. Particularly, the optimized fourth-order algorithm compared with the mixed leapfrog scheme provides a good precision and needs no expensive additional computational time. The chaoticity of the system can lead to fast iterative convergence and improve the computational efficiency. Because symplectic integrators have no secular change in the energy errors, can give more reliable dynamical information from gravitational waves. See Phys. Rev. D 82, 124040 (2010) for more information.

In sum, we have confirmed that the dynamics of the spinning compact binaries can not be determined uniquely by the dynamical parameters, initial conditions, and initial spin angles. Instead, a combination of them is a sourse for causing chaos. These support the results of Wu and Xie (Phys. Rev. D 77, 103012 (2008)). Finally, the gravitational waveforms from chaotic orbits are proved to be stochastic.