

## CONFIRMATION OF 8 PLANETS IN 4 MULTI-PLANET SYSTEMS VIA TTVS IN 1350 DAYS

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## ABSTRACT

Analysis of the transit timing variations (TTVs) of candidate pairs near mean-motion resonances (MMRs) is an effective method to confirm planets. Hitherto, 66 planets in 33 multi-planet systems have been confirmed via TTVs. We analyse the TTVs of all candidates from the most recent *Kepler* data with a time span as long as about 1350 days (Q0-Q15). The anti-correlations and mass upper limits of candidate pairs in the same system are calculated, using an improved method suitable for long-period TTVs. If the false alarm probability (FAP) of a candidate pair is less than  $10^{-3}$  and the mass upper limit for each candidate is less than  $13 M_J$ , we confirm them as planets in the same system. Finally, 8 planets in 4 multi-planet systems are confirmed via analysis of their TTVs. All of the 4 planet pairs are near first-order MMRs, including KOI-2672 near 2:1 MMR, KOI-1236, KOI-1563 and KOI-2038 near 3:2 MMR. Four planets have relatively long periods ( $> 35$  day). KOI-2672.01 has a period of 88.51658 day and a mass upper limit about  $17.086 M_{\oplus}$ . It is the second longest periodical planet confirmed via TTVs.

*Subject headings:* methods: data analysis — techniques: photometric — planets and satellites: detection — stars: individual (KID 6677841/KOI-1236, KID 5219234/KOI-1563, KID 8950568/KOI-2038, KID 11253827/KOI-2672)

## 1. INTRODUCTION

*Kepler* is a landmark space telescope designed to search for Earth-sized planets in and near the habitable zone of Sun-like stars (Borucki et al. 2010). A transit signature can be detected by *Kepler* owing to its high photometric precision of 20 ppm (Koch et al. 2010). *Kepler* has made significant achievements since it was launched on March 6, 2009. 1235 candidates were identified using the first four months of data (Borucki et al. 2011) and it outnumbered 2300 as the observation time increased to 16 months (Batalha et al. 2013). By now 138 planets have been confirmed by *Kepler*. Recently a new set of data have been released (Q0-Q15 from MAST, <http://archive.stsci.edu/kepler>). The observational duration of *Kepler* is extended to  $\sim 1,350$  day. The quantities of all candidates have reached as many as 3548, including 1475 candidates in multiple systems (up to July, 2013). Due to longer observation time, more long-period candidates are found, e.g. KOI-3946.01 with a period of 308.545 day. Some single transiting systems are changed to multiple systems, e.g. KOI-255.02.

For multi-planet systems, the interactions between the planets will cause variations of their transit midtimes (Agol et al. 2005; Holman & Murray 2005). *Kepler*-9 was confirmed as a multi-planet system via transit timing variations (TTVs) successfully by Holman et al. (2010). Even nontransiting planets can be detected via TTVs (Ballard et al. 2011; Nesvorný et al. 2012). TTVs for a two-planet system will be anti-correlated because of the conservation of energy. By recognizing such anti-correlation and computing the amplitudes of their TTVs, we can infer the mass upper limits of these candidates and therefore confirm them as planets (Steffen et al. 2013; Xie 2012). A series of papers have described the theory in detail by *Kepler* group. They have confirmed

48 planets in 23 systems combining the anti-correlation method and the dynamical simulation of orbital stability (Ford et al. 2012; Steffen et al. 2012a; Fabrycky et al. 2012a; Steffen et al. 2013). Xie (2012) also confirmed 12 planet pairs via TTVs. The masses of 3 planet pairs are exactly calculated while the mass upper limits of others are inferred via the amplitudes of their TTVs.

Hitherto, all confirmed planet pairs via TTVs are less than 800 days. Although some planets with long periods can be confirmed via TTVs, e.g. *Kepler*-30 d with a period about 143.2 day (Fabrycky et al. 2012a), confirmations of these planets are less convinced and their masses are quite uncertain due to few transit midtimes in a few hundred of days. In this paper, we utilize the recent data realised on July 1st, 2013 (Q0-Q15). With a time span as long as 1350 day, we can search for planet pairs near the first order MMRs with long-period TTVs. Their mass upper limits can be inferred via the amplitude described by Lithwick et al. (2012).

The arrangement of this paper is as follows. We interpret how to obtain TTVs of KOI candidates and describe the confirmation method in Section 2, i.e. recognizing anti-correlations, calculating false alarm probabilities (FAPs) and estimating mass limits. The properties of the 8 confirmed planets in 4 systems are listed and compared with planets confirmed via TTVs previously in section 3. We summarize and discuss our results in section 4.

## 2. TTV ANALYSIS METHOD

We use the *Kepler* data in long cadence (LC) to calculate TTVs. We analyse all *Kepler* objects of interest (KOIs) flagged as "CANDIDATE". We can obtain the corresponding orbital parameters and stellar properties from NASA Exoplanet Archive (<http://exoplanetarchive.ipac.caltech.edu/> Akesson et al. 2013).

We compute TTVs for all candidates following the steps described by Xie (2012). But some improvements are made. We do not eliminate contaminated segments for KOIs with periods longer than one month. Because they usually have long transit durations. The segments with a four times of the duration are inevitably easy to be polluted. Besides, we detrend the outside of every segment with one to three order polynomials. The best fit with the lowest chi-square is adopted. Finally we obtain TTVs for all multiple systems. Since *Kepler* data have a high precession of 20 ppm, we can verify TTVs of several minutes.

To confirm planet pairs in a multiple system, two criterions must be satisfied. Firstly they must be anti-correlated so that we can infer that the two candidates are in the same system. The anti-correlation can exclude the conditions that the transits are caused by some external reasons, e.g. background eclipsing binaries or background stars transited by planets, etc. To estimate the probability that the observed anti-correlation is caused by random fluctuations, we also check the false alarm probabilities (FAPs) for candidate pairs. Only with a low FAP of less than  $10^{-3}$ , the candidate pairs are considered to interact with each other reliably. Secondly the physical parameters of the candidate pairs must be in acceptable ranges, especially their masses must be less than  $13 M_J$  (Spiegel et al. 2011). We are interested in candidate pairs near first-order MMRs, namely, the period ratio  $P_2/P_1$  for each pair is around  $j : j - 1$ .  $P_1$  and  $P_2$  are the periods of the inner and outer candidate. We use a normalized distance to resonance parameter  $\Delta$  to select out such candidate pairs.  $\Delta$  is defined as

$$\Delta = \frac{P_2}{P_1} \frac{j-1}{j} - 1. \quad (1)$$

All confirmed planets via TTVs near first-order MMRs satisfy  $|\Delta| < 0.06$  as shown in Figure 6(b). We adopt this threshold to select candidate pairs. We will check the anti-correlated periods of the adopted candidate pairs, calculate their FAPs and set a threshold of  $10^{-3}$ , and estimate their mass upper limits via TTV amplitudes. These steps will be described in detail in the following subsections.

### 2.1. TTV Anti-correlations and FAPs

TTV anti-correlation can indicate the perturbations between two planets in the same system because of the conservation of energy. Transit midtimes of the two planets will vary simultaneously and oppositely. Many multiple systems show such characteristic. The anti-correlation method has been described firstly by Steffen et al. (2012a). We improve it by considering the linear trend mixed in long-period TTVs. When computing TTVs a simple linear fitting of transit midtimes will be applied. However it will not remove the secular trend of long-period TTVs. Although the linear trend of TTVs does not affect the anti-correlation tendency, it will affect TTV amplitude and cause an inaccurate estimation of mass upper limit. Thus we fit TTVs for each KOI using the following function:

$$f = A \sin\left(\frac{2\pi t}{P}\right) + B \cos\left(\frac{2\pi t}{P}\right) + Ct + D \quad (2)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are model parameters with test period  $P$ . We vary  $P$  with a wide range from 100 to 1500 day to get a series of  $A$ ,  $B$ ,  $C$  and  $D$  together with their uncertainties  $\sigma_A$ ,  $\sigma_B$ ,  $\sigma_C$  and  $\sigma_D$ . An anti-correlation parameter  $\Xi(P)$  is calculated as

$$\Xi(P) = -\left(\frac{A_1 A_2}{\sigma_{A_1} \sigma_{A_2}} + \frac{B_1 B_2}{\sigma_{B_1} \sigma_{B_2}}\right) \quad (3)$$

where the subscripts "1" and "2" represent the two planets. The maximum value  $\Xi_{\max}$  represents the strongest anti-correlation among all the test periods. We select out candidate pairs with strong and significant anti-correlations by ranking their  $\Xi_{\max}$ . Only the top fifty remain.

FAPs are checked for these candidate pairs. For each pair, we scramble their TTVs randomly and obtain a similar  $\Xi'_{\max}$ . We use a superscript of " ' " to distinguish it from  $\Xi_{\max}$  which is obtained from the nominal data. The same process repeats  $10^4$  times. The proportion of  $\Xi'_{\max}$  larger than  $\Xi_{\max}$  represents the degree of FAP. Only candidate pairs with  $\text{FAP} < 10^{-3}$  are accepted (Steffen et al. 2012a).

### 2.2. Mass Upper Limit

Masses and free eccentricities of planet pairs near (but not in) first-order MMRs can be estimated via TTV amplitudes (Lithwick et al. 2012; Xie 2012). We will estimate the mass upper limits of the accepted candidate pairs theoretically instead of simulating their stability as done by Kepler group.

For a system near  $j : j - 1$  MMR, The amplitudes of a planet pair are (Lithwick et al. 2012);

$$|V_1| = P_1 \mu_2 \left| \frac{f}{\Delta} \right| \frac{1}{\pi j^{2/3} (j-1)^{1/3}} \quad (4)$$

$$|V_2| = P_2 \mu_1 \left| \frac{g}{\Delta} \right| \frac{1}{\pi j} \quad (5)$$

where  $V$  is the TTV amplitude,  $P$  is the orbital period,  $\mu$  is the ratio of the candidate to the star, subscripts "1" and "2" represent the inner and outer candidate,  $g$  and  $f$  are Laplace coefficients.

In equation (4) and equation (5) we have set  $|Z_{free}| = 0$  to get the mass upper limit.  $Z_{free}$  is a parameter related to the complex eccentricities of the two planets. We use the model parameters corresponding to  $\Xi_{\max}$  to calculate TTV amplitudes. Resonance order can be obtained on the basis of the ratio of the periods.

Before we calculate the mass limit, we check the anti-correlated period  $P_{\text{anti}}$  and the theoretical synodic period  $P_{\text{syn}}$  for each pair.  $P_{\text{anti}}$  corresponds to  $\Xi_{\max}$ .  $P_{\text{syn}}$  for a system near  $j : j - 1$  MMR can be obtained by

$$P_{\text{syn}} = \frac{1}{\left| \frac{j}{P_2} - \frac{j-1}{P_1} \right|} \quad (6)$$

For a multiple system with only two candidates,  $P_{\text{anti}}$  should be equal to  $P_{\text{syn}}$  under ideal conditions. However, for systems containing more than two candidates or observed TTVs with large errors, the two periods will not be equal exactly. Especially for long-period candidates with limited TTV samples, the  $P_{\text{anti}}$  will be less accurate. When we compute mass upper limits we will use  $P_{\text{syn}}$  instead of  $P_{\text{anti}}$  to avoid its large error.

### 3. CONFIRMATION OF 8 PLANETS IN 4 SYSTEMS

We confirm 8 planets in 4 KOI systems. They are all near first-order MMRs, including one pair near 2:1 MMR (KOI-2672) and three pairs near 3:2 MMR (KOI-1236, KOI-1563 and KOI-2038). We compute  $P_{\text{syn}}$  of the four planet pairs. The results are listed in Table 1. As pointed in section 2.2, we can see that KOI-1563 and KOI-2672 have planet pairs with almost the same  $P_{\text{anti}}$  and  $P_{\text{syn}}$ , while KOI-1236 and KOI-2038 does not. Their  $P_{\text{anti}}$  and  $P_{\text{syn}}$  will be analysed combining their frequency spectrums in the following. We will describe these planet pairs individually.

#### 3.1. KOI-1236.01 and 1236.03

KOI-1236 has three candidates. Moving around a star with  $1.27 R_{\odot}$ . Radius of the three candidates are  $4.3 \pm 1.8$ ,  $2.6 \pm 1.1$  and  $3.1 \pm 1.3 R_{\oplus}$  respectively. The periods of KOI-1236.01 and KOI-1236.03 are 35.74113 and 54.3995 days near 3:2 MMR. The inner most KOI-1236.02 have a period of 12.309717 days.

Figure 1(a) and Figure 1(d) illustrates distinct anti-correlation between KOI-1236.01 and 1236.03 with a low FAP=0.0006 (see Table 1). However their  $P_{\text{syn}}$  is unequal to  $P_{\text{anti}}$  as shown in Figure 1(c). Combining the frequency spectrums of all the candidates in this system, we can see only KOI-1236.01 and KOI-1236.03 consist with the theoretical synodic periods  $P_{\text{syn}} \sim 1234$  d well, while KOI-1236.02 has much smaller powers. Therefore, it can be inferred that KOI-1236.01 and KOI-1236.03 interact with each other obviously. The perturbations of the innermost KOI-1236.02 are very limited on both KOI-1236.01 and 1236.03. Fortunately, although the synodic period is as long as  $\sim 1234$  d, it is still less than our observational duration.

We fit the TTV amplitudes of KOI-1236.01 and KOI-1236.03 with the theoretical  $P_{\text{syn}}$  via equation (2) to obtain their mass upper limits. Amplitudes of 91.585 min for KOI-1236.01 and 191.462 min for KOI-1236.03 are fitted here. Their corresponding residuals are 19.245 min and 44.983 min. The residuals might be caused by the perturbations of KOI-1236.02. According to equation (4) and (5), we obtain the mass upper limits of  $61.667 M_{\oplus}$  for KOI-1236.01 and  $48.597 M_{\oplus}$  for KOI-1236.03. Considering the large radius compared with rock planet, these two planets are not likely to be rock planets although their maximum densities shown in Table 2 are similar with rock planet.

#### 3.2. KOI-1563.01 and 1563.02

Four candidates are moving around star KOI-1563 with  $0.87 R_{\odot}$ . Some properties of the host star are listed in Table 3. Radius of the four candidates are  $3.60 \pm 1.30$ ,  $3.30 \pm 1.10$ ,  $2.16 \pm 0.77$  and  $3.70 \pm 1.30 R_{\oplus}$  respectively. The periods of KOI-1563.01 and KOI-1563.02 are 5.487006 and 8.29113 days, which are near 3:2 MMR. Other candidates have periods of 3.205322 days for KOI-1563.03 and 16.73826 days for KOI-1563.04.

Figure 2(a) and Figure 2(d) shows that KOI-1563.01 and KOI-1563.02 have significant anti-correlation with a low FAP=0.0002 (see Table 1). Their  $P_{\text{syn}}$  is approximately equal to  $P_{\text{anti}}$  as shown in Figure 2(c). Therefore, it can be inferred that the perturbations of KOI-1563.03 and KOI-1563.04 are limited on both KOI-1563.01 and

KOI-1563.02.

To obtain the mass upper limits of these two planets, we fit the TTV amplitudes of KOI-1563.01 and KOI-1563.02 with theoretical  $P_{\text{syn}}$  via equation (2). We have almost three complete synodic cycles during the observation time. Therefore we can obtain a very accurate result. An amplitude of 6.675 min for KOI-1563.01 and 12.704 min for KOI-1563.02 are fitted here. The corresponding residuals are 3.604 min and 7.608 min. Since KOI-1563.02 and 1563.04 are near 2:1 MMR with a small  $\Delta = 0.009$ , we can infer that the mass of the outermost candidate is less than  $7.570 M_{\oplus}$  if it is responsible for the large residuals. According to Eq.(3) and (4), we obtain the mass upper limits of  $8.963 M_{\oplus}$  for KOI-1563.01 and  $7.664 M_{\oplus}$  for KOI-1563.02. These two planet seems to be super-Earth according to their radius. However, the densities of the planet pair are relatively low as listed in Table 2, and therefore they might contain a lot of gas or some other light materials.

#### 3.3. KOI-2038.01 and 2038.02

Star KOI-2038 has four candidates. some properties of this star are listed in Table 3. Radius of the candidates are  $1.99 \pm 0.86$ ,  $2.20 \pm 0.95$ ,  $1.56 \pm 0.68$  and  $1.61 \pm 0.70 R_{\oplus}$  respectively. The periods of KOI-2038.01 and KOI-2038.02 are 8.305992 and 12.51217 days, which are near 3:2 MMR. Other candidates have periods of 17.91304 days for KOI-2038.03 and 25.21767 days for KOI-2038.04.

Figure 3(a) and Figure 3(d) shows that KOI-2038.01 and KOI-2038.02 have significant anti-correlation with a low FAP  $< 10^{-4}$  (see Table 1). However their  $P_{\text{syn}}$  is unequal to  $P_{\text{anti}}$  as shown in Figure 3(c). Combining the frequency spectrums of all the candidates in this system, we can see only KOI-2038.01 and KOI-2038.02 consist with  $P_{\text{syn}} \sim 977$  d the most while others have much smaller powers. Therefore, it can be inferred that KOI-2038.01 and KOI-2038.02 interact with each other obviously. The perturbations of KOI-2038.03 and KOI-2038.04 are very limited on both of them.

To obtain the mass upper limits of these two planets, we fit the TTV amplitudes of KOI-2038.01 and KOI-2038.02 with the theoretical  $P_{\text{syn}}$  via equation (2). An amplitude of 40.541 min for KOI-2038.01 and 51.382 min for KOI-2038.02 are fitted here. The corresponding residuals are 11.206 min and 14.850 min. The perturbations of KOI-2038.03 and 2038.04 may be the main reason for the residuals. Assuming the residuals of KOI-2038.02 are mainly caused by KOI-2038.03, the mass of KOI-2038.03 must be less than  $42.457 M_{\oplus}$ . Otherwise, if the residuals of KOI-2038.02 are mainly caused by KOI-2038.04, the mass of KOI-2038.04 must be less than  $8.579 M_{\oplus}$ . According to equation (4) and (5), we obtain the mass upper limits of  $14.789 M_{\oplus}$  for KOI-2038.01 and  $18.890 M_{\oplus}$  for KOI-2038.02. Table 2 show the densities of these two planets, which seems as twice large as the density of Earth.

#### 3.4. KOI-2672.01 and 2672.02

KOI-2672 with a radius of  $1.04 R_{\odot}$ , has only two candidates around. Other parameters of KOI-2672 are listed in Table 3. Radius of the two candidates are  $5.3 \pm 2.1$  and  $3.5 \pm 1.4 R_{\oplus}$  respectively. The periods of KOI-2672.01 and KOI-2672.02 are 88.51658 and 42.99066 days, which are near 3:2 MMR.



Figure 4(a) and Figure 4(d) shows that KOI-2672.01 and KOI-2672.02 have significant anti-correlation with a low  $FAP < 10^{-4}$  (see Table 1). Their  $P_{\text{syn}}$  is approximately equal to  $P_{\text{anti}}$  as shown in Figure 4(c). The strongest powers in their frequency spectrums also consist with  $P_{\text{syn}}$ . Although the synodic period is as long as  $\sim 1500$  d, our observational duration still covers almost one cycle.

To obtain the mass upper limits of KOI-2672.01 and 2672.02, we fit their TTV amplitudes with theoretical  $P_{\text{syn}}$  via equation (2). An amplitude of 77.687 min for KOI-2672.01 and 28.952 min for KOI-2672.02 are fitted here. According to equation (4) and (5), we obtain the mass upper limits of  $17.086 M_{\oplus}$  for KOI-2672.01 and  $80.133 M_{\oplus}$  for KOI-2672.02. Our best fitting via equation (2) produce a residuals about 3.394 min for KOI-2672.01 and 3.115 min for KOI-2672.02. The small residuals indicate that there is no large perturbations in this system. Assuming another planet outside is near 2:1 MMR with KOI-2672.01, we infer that the mass of the assumed planet must be less than  $1.718 M_{\oplus}$  from Equation (4). The density of the planet pair are presented in Table 2, which show a large differences between them. This might give an evidence that these two planets are formed in quite different locations around the host star.

### 3.5. Comparison with Confirmed Kepler Planets

We have confirmed 8 planets via TTVs and list the parameters of them in Table 2. Due to the long period of their TTVs, we have found a planet with a period of 88.51658 days, the second longest periodical planet confirmed via TTVs under Neptune size, as shown in Figure 5. The longest one is Kepler-30 d, which are near 5:2 MMR with Kepler-30 c and confirmed by only four data and the outline of its TTV is hardly fitted (Fabrycky et al. 2012a). Thus KOI-2672.01 is the longest periodical planet near first-order MMR. We also note that this system has only two planets detected.

We statistic all the Kepler planets near MMRs as shown in Figure 6 and compare our results with them. 66 planets are confirmed via TTVs previously. Adding one pair near 2:1 MMR and three pairs near 3:2 MMR, there are totally 20 pairs near 2:1 MMR and 17 pairs near 3:2 MMR. Only 9 pairs are in other first-order MMRs and 10 pairs in the second-order MMRs. Theoretical works pointed that coplanar planet pairs prefer to stay in or near 2:1 MMR (Wang et al. 2012). However, a large fraction of planet pairs stay near 3:2 MMR. It's very interesting because 2:1 seems to have a wider resonance width and planet pairs must pass through the 2:1 MMR and approach to 3:2 MMR. Migration theories must interpret this fact in multi-planet systems near 2:1 and 3:2 MMRs. Figure 6 present  $\Delta$  defined as equation (1) and the mass ratio of planet pair in each system. It's obvious that 75% planet pairs have a period ratio that are a little larger than the exact value  $j : (j - 1)$ . This is also revealed by Fabrycky et al. (2012b) and interpreted by (Lithwick & Wu 2012) via tidal dissipation.

We have found 4 planet pairs near first-order MMRs via TTVs with a time span as long as  $\sim 1350$  days, i.e., KOI-1236.01 and KOI-1236.03 near 3:2 MMR, KOI-1563.01 and KOI-1563.02 near 3:2 MMR, KOI-2038.01 and KOI-2038.02 near 3:2 MMR and KOI-2672.01 and KOI-2672.02 near 2:1 MMR. We also estimate their mass upper limits via Equation (4) and (5), as shown in Table 2. KOI-2672 has only two planets with a long theoretical  $P_{\text{syn}}$  about 1500 d, and the outer planet has the longest period among the planet pairs near first-order MMRs. Considering the large difference between their densities, we infer that they formed in different regions and evolved in current stage due to migration or some other mechanisms.

We also compare our new confirmed planets with other Kepler planets, especially these planet pairs near MMRs. Adding the new four pairs near MMRs, we find that about 36% and 30% of the planet pairs stay near 2:1 and 3:2 MMR, respectively. A little more planet pairs stay near 2:1 MMR than those near 3:2 MMR. This is very critical to constrain the migrating rate of planets during the evolution of multi-planet systems (Ogihara & Kobayashi 2013). Planet pairs are prefer to keep an architecture that the outside planets are a little outside than the exact positions with period ratios of  $j : j - 1$ , which is consistent with (Fabrycky et al. 2012b).

In this paper, we only consider planet pairs near first-order MMRs. To estimate the influences on TTVs of planet pairs near higher order MMRs, the equations of TTVs are much more complex because the evolution of their eccentricities are coupling with each other and time-dependent. There is no analytical resolution without any assumptions even in the order of  $ecc^2$ . Additionally, planet pairs near higher order MMRs have less influences on each other than first-order MMRs, thus it is much hard to estimate their mass upper limits. However, the anti-correlations of their TTVs and small FAPs are also available to confirm these two gravitationl objects. The fraction of planet pairs near high order MMRs are relatively rare, only 10 pairs are confirmed in the observation of *Kepler*.

Here we use the TTVs spanning up to about 1350 days. TTVs in a longer timescale can provide more information about the secular influences in planet pairs near or even in MMRs (Ketchum et al. 2013; Boué et al. 2012). Subsequent observations of transiting planets or *Kepler* candidates provide us more information about their TTVs to investigate the dynamical properties of multi-planet systems and might confirm planets with longer period in habitable zone.

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## 4. SUMMARY AND DISCUSSION

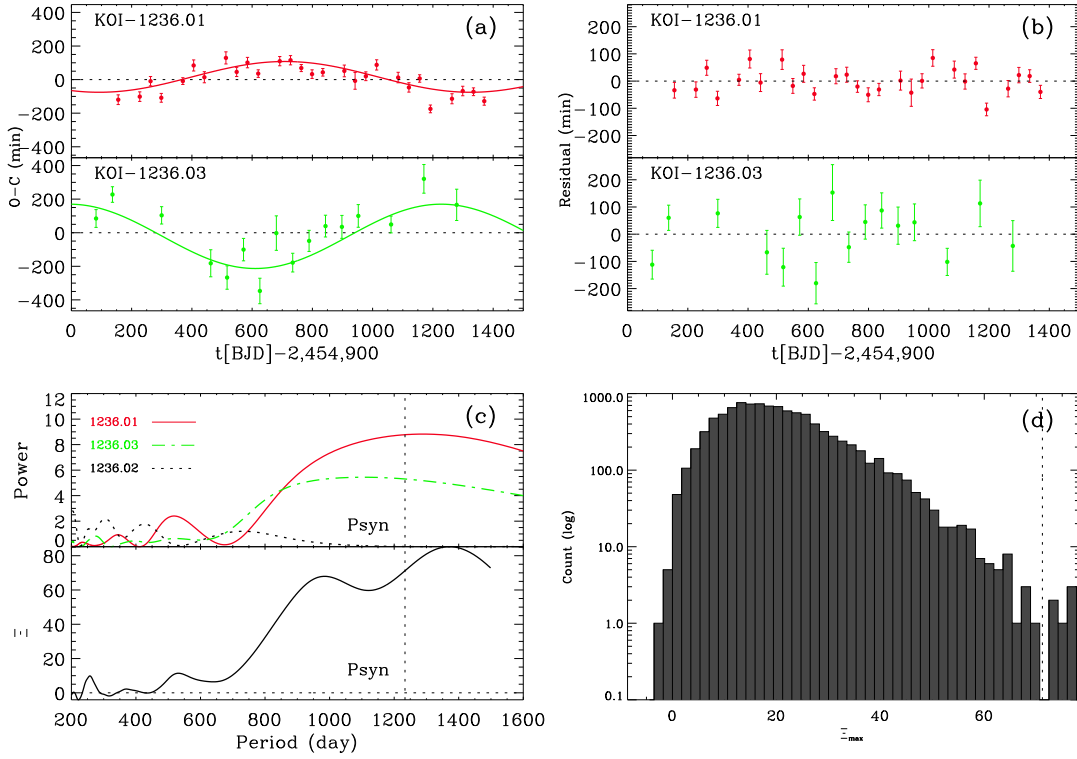


FIG. 1.— (a) TTVs and the best-fit results for KOI-1236.01 and KOI-1236.03. Points with errorbars show TTVs. Solid lines correspond to the best-fit models using their theoretical synodic period ( $P_{syn} = 1234.0$  day). (b) Residuals between the TTVs and the best-fit results for KOI-1236.01 and KOI-1236.03. (c) The frequency spectrums for all candidates in KOI-1236 (upper panel) and the anti-correlation curve with different test periods (lower panel). The vertical dashed line corresponds to  $P_{syn}$ . (d) The Monto Carlo test result of FAP. The vertical dashed line corresponds to  $\Xi_{max}$  calculated using  $P_{syn}$ . The FAP for KOI-1236.01 and KOI-1236.03 is 0.0006.

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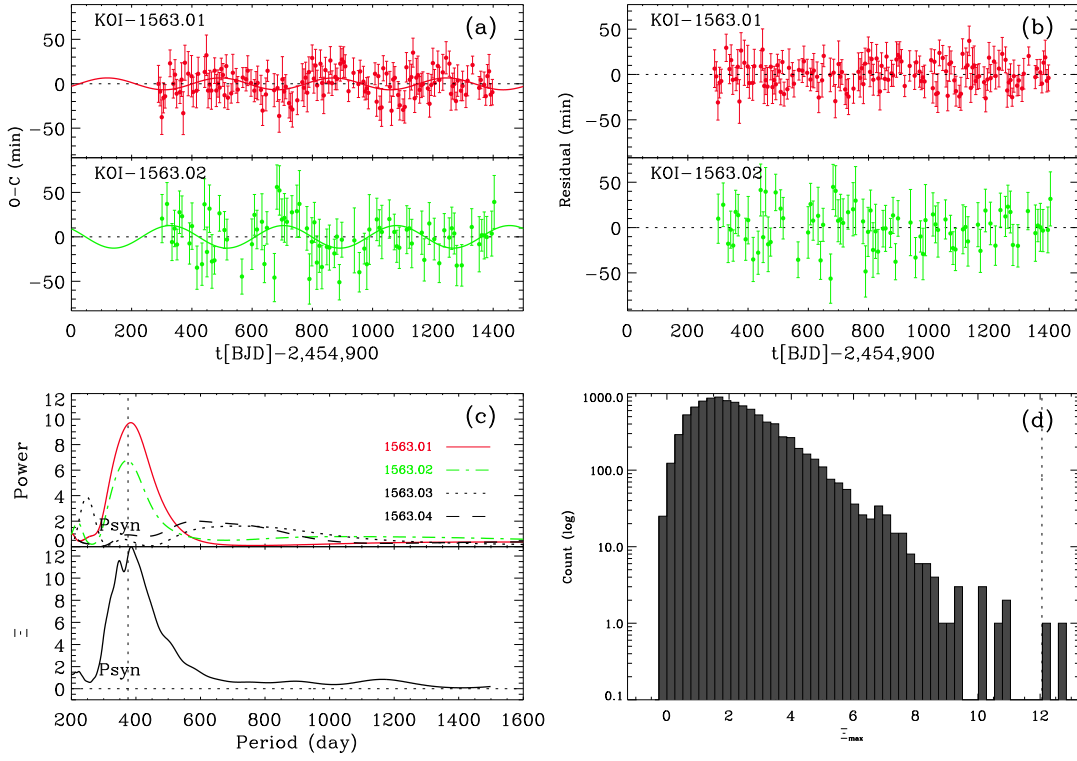


FIG. 2.— Similar as Figure 1 but for KOI-1563.01 and KOI-1563.02. Their  $P_{\text{syn}} = 375.2$  day and FAP is 0.0002.

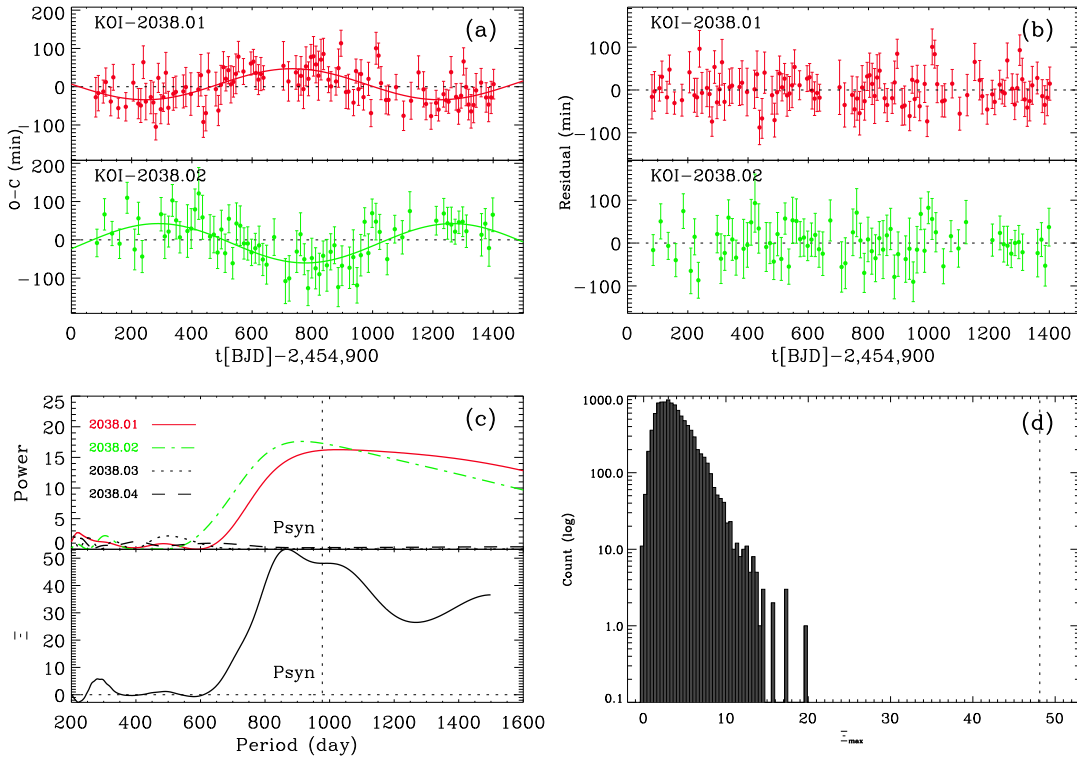


FIG. 3.— Similar as Figure 1 but for KOI-2038.01 and KOI-2038.02. Their  $P_{\text{syn}} = 977.1$  day and FAP is less than  $10^{-4}$ .

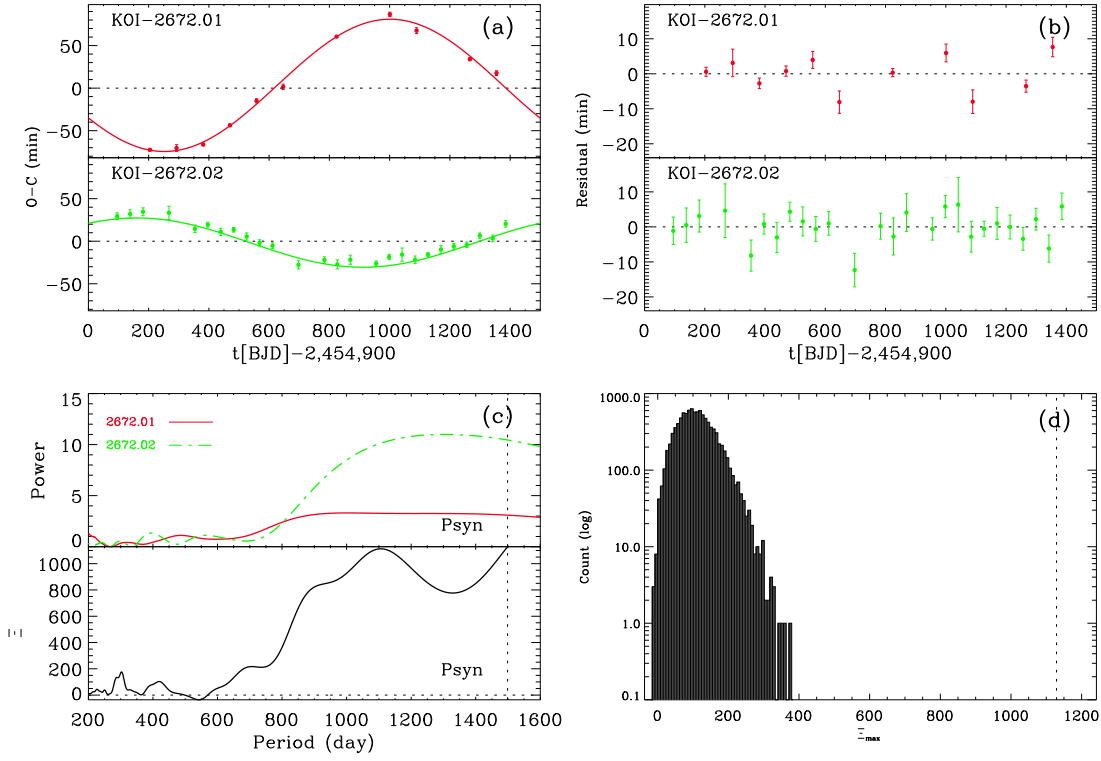


FIG. 4.— Similar as Figure 1 but for KOI-2672.01 and KOI-2672.02. Their  $P_{\text{syn}} = 1501.0$  day and FAP is less than  $10^{-4}$ .

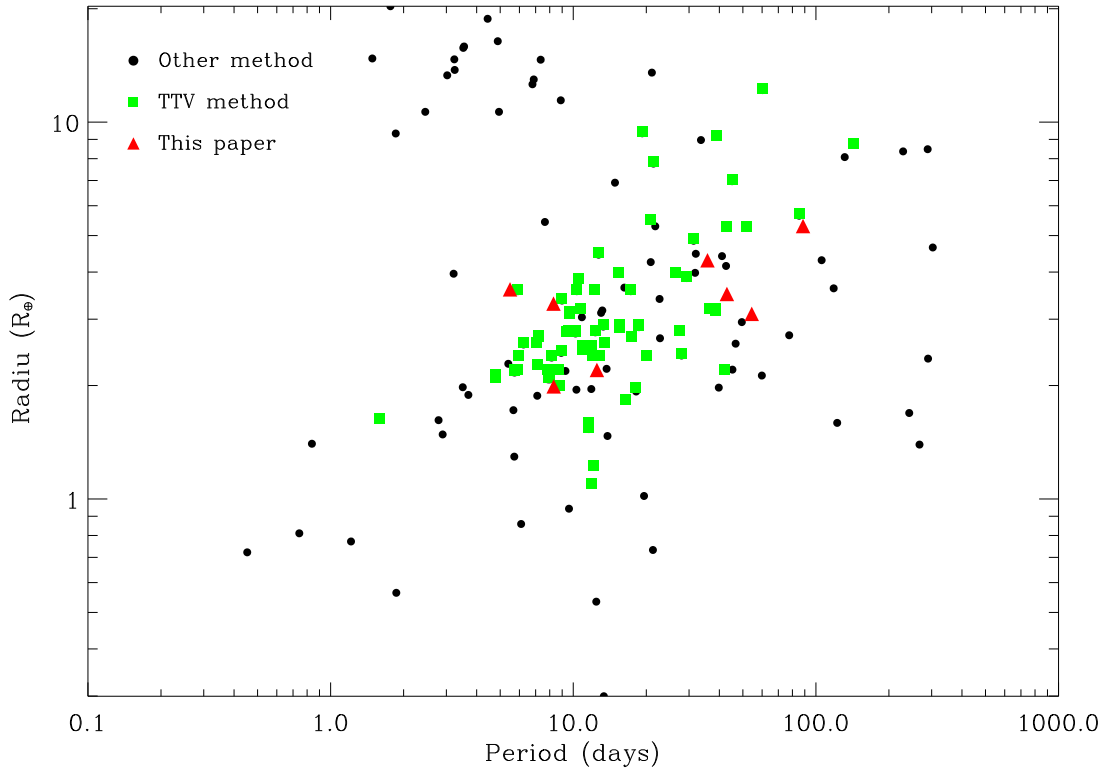


FIG. 5.— Period vs. radius of *Kepler* planets. Black filled circles are confirmed via RV or BLEND method. Green squares are confirmed via TTV method. Red triangles are confirmed in this paper, including four planets with relatively long periods ( $> 35$  day).

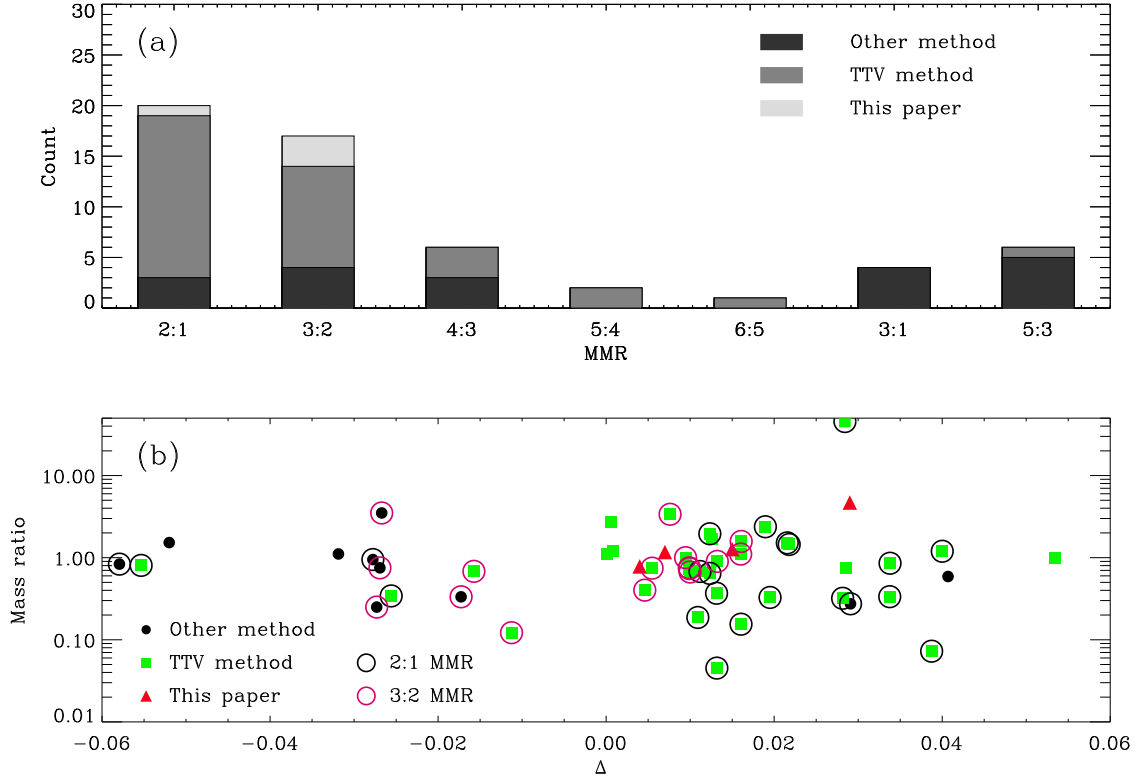


FIG. 6.— (a) Distribution of the MMRs of *Kepler* planet pairs. Dark grey are confirmed via RV or BLEND method. Grey are confirmed via TTV method. Light grey are confirmed in this paper. In this paper we add one pair near 2:1 MMR and 3 pairs near 3:2 MMR. (b) Parameter  $\Delta$  (see equation(1)) vs. mass ratio in first-order MMRs of *Kepler* planets.  $\Delta$  of most planet pairs are great than 0, which infers that the outer planets are prefer to stay a little far away from the exact first-order MMR position (Fabrycky et al. 2012b).



TABLE 1  
ANTI-CORRELATIONS AND FAPs OF 8 CANDIDATES IN 4 SYSTEMS

KOI	Anti-correlation	FAP	$P_{anti}$ (days)	$P_{syn}$ (days)	$P_j/P_{j-1}$	$\Delta$
1236	KOI1236.01 - KOI1236.03	0.0006	1498.7	1234.0	0.657	0.015
1563	KOI1563.01 - KOI1563.02	0.0002	384.6	375.2	0.662	0.007
2038	KOI2038.01 - KOI2038.02	<0.0001	863.0	977.1	0.664	0.004
2672	KOI2672.01 - KOI2672.02	<0.0001	1449.3	1501.0	0.486	0.029

TABLE 2  
MAIN PROPERTIES OF CONFIRMED PLANETS.

KOI	$T_0$ (BJD-2,454,900)	Period (days)	$\sigma_{\text{Period}}$ (days)	a (AU)	$T_{eq}$ (K)	Duration (hr)	Radius $R_{\oplus}$	Mass <sub>max</sub> $M_{\oplus}$	$\sigma_{\text{Mass}_{\text{max}}}$ $M_{\oplus}$	$\rho_{max}$ g/cm <sup>3</sup>
1236.01	84.06120	35.741130	0.000190	0.232	699	8.1417	4.30	61.667	14.488	4.337
1236.03	81.87990	54.399500	0.000630	0.307	607	7.7780	3.10	48.597	10.212	8.467
1563.01	289.07960	5.487006	0.000059	0.059	833	2.9700	3.60	8.963	5.368	1.075
1563.02	292.62760	8.291130	0.000130	0.077	729	3.2008	3.30	7.664	4.138	1.209
2038.01	72.71660	8.305992	0.000093	0.079	814	3.8461	1.99	14.789	4.274	10.202
2038.02	72.45610	12.512170	0.000140	0.103	712	4.3714	2.20	18.890	5.221	9.679
2672.01	115.65259	88.516580	0.000180	0.369	413	6.8748	5.30	17.086	1.838	0.643
2672.02	95.51176	42.990660	0.000130	0.228	525	4.7047	3.50	80.133	3.501	10.115

TABLE 3  
STELLAR PROPERTIES OF CONFIRMED MULTI-PLANET SYSTEMS.

KOI	KIC	Kp (mag)	Teff (K)	log(g) (cm/s <sup>2</sup> )	R* ( $R_{\odot}$ )	M* ( $M_{\odot}$ )	RA (J2000)	DEC (J2000)
1236	6677841	13.659	6779	4.35	1.27	1.31	19 09 33.889	+42 11 41.40
1563	5219234	15.812	4918	4.51	0.87	0.89	19 56 53.840	+40 20 35.46
2038	8950568	14.779	5666	4.57	0.84	0.95	19 23 53.621	+45 17 25.16
2672	11253827	11.921	5565	4.33	1.04	0.84	19 44 31.875	+48 58 38.65